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TECHNICAL NOTE 3888

SIMPLIFIED METHOD FOR ESTIMATING COMPRESSIBLE LAMINAR
HEAT TRANSFER WITH PRESSURE GRADIENT

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SIMPLIFIED METHOD FOR ESTIMATING COMPRESSIBLE LAMINAR HEAT

TRANSFER WITH PRESSURE GRADIENT

By Eli Reshotko

SUMMARY

In the present report an approximation is made in the method of Technical Note 3326, which simplifies the calculation of heat transfer. Good agreement with the method of Technical Note 3326 is expected for isothermal surfaces with adverse and small favorable pressure gradients regardless of surface temperature and also for flow with large favorable pressure gradient over highly cooled surfaces.

INTRODUCTION

A major factor influencing the design of vehicles for supersonic and hypersonic flight is aerodynamic heating. While for simple shapes such as the flat plate, the cone, and stagnation points the amount of laminar heat transfer may be estimated using simple formulas, such is not generally the case for a calculation with arbitrary pressure gradient.

An integral method for calculating compressible laminar boundary layers with heat transfer and pressure gradient is given in reference 1. This method is an adaptation of the correlation technique of Thwaites (ref. 2) to the compressible boundary layer using the exact solutions of reference 3. The present report points out an approximation which simplifies the calculation of laminar heat transfer. Examples are presented to illustrate the approximate technique, and numerical comparison is made with the complete method of reference 1. Some approximate techniques for slender bodies introduced by Jack and Diaconis (ref. 4) are described in conjunction with the examples.

SYMBOLS

- A coefficient in eqs. (2) and (3)
- B exponent of Mach number in eqs. (2) and (3)(fig. 1)

C_f	local skin-friction coefficient, $\tau_w / \frac{1}{2} \rho_w u_e^2$
$\frac{C_f Re_w}{Nu}$	Reynolds analogy parameter (ref. 1)
C_p	pressure coefficient, $\frac{p - p_\infty}{\frac{1}{2} \gamma p_\infty M_\infty^2}$
c_p	specific heat at constant pressure
D	throat diameter of rocket nozzle (example IV)
h	heat-transfer coefficient
K	$\frac{3\gamma - 1}{2(\gamma - 1)}$ ($K = 4$ for $\gamma = 1.4$)
k	thermal conductivity
L	arbitrary length
z	dimensionless shear parameter (ref. 1), $\frac{\theta}{u_e} \frac{t_w}{t_e} \left(\frac{\partial u}{\partial y} \right)_w$
M	Mach number
Nu	Nusselt number, $\frac{hx}{k_w}$
n	correlation number (ref. 1), $\frac{-\left(\frac{du_e}{dx}\right) \theta^2}{v_w} \left(\frac{t_w}{t_e} \right)^2 \left(\frac{t_0}{t_e} \right)$
P'	dimensionless pressure gradient, $\frac{L}{p_e} \frac{dp_e}{dx} / \gamma M_e^2$
Pr	Prandtl number, $\frac{\mu c_p}{k}$ (evaluated at wall temperature)
p	pressure
R	radius of axially symmetric body
Re_w	Reynolds number, $\frac{\rho_w u_e x}{\mu_w}$

$$r/s_w = \frac{n\theta t_w [\partial y(t/t_e)]_w}{t_w - t_0}$$

t temperature

u_∞ external velocity

x coordinate along surface

y coordinate normal to surface

α exponent of Prandtl number for heat-transfer modification

γ ratio of specific heats

θ boundary-layer momentum thickness

μ absolute viscosity

ν kinematic viscosity

ρ density

τ_w shear stress at wall, $\mu_w \left(\frac{\partial u}{\partial y} \right)_w$

Subscripts:

cyl cylinder

e local flow outside boundary layer (external)

w wall or surface value

∞ free-stream static value

0 free-stream stagnation value

CALCULATION PROCEDURE

The calculation of laminar heat transfer over isothermal surfaces by the method of reference 1 is first summarized and then the approximation introduced.

The heat-transfer parameter $Nu/\sqrt{Re_w}$ for Prandtl number 1 from equations (36) and (37) of reference 1 is

$$\left(\frac{Nu}{\sqrt{Re_w}} \right)_{Pr=1} = \frac{2l}{\left(\frac{C_f Re_w}{Nu} \right)_{Pr=1}} \sqrt{\frac{\frac{x}{L}}{\frac{n}{P' \frac{t_0}{t_e}}}} \quad (1)$$

The factor $n/P' \frac{t_0}{t_e}$, which is directly proportional to the square of momentum thickness, is obtained in reference 1 by quadrature from the momentum integral equation. For two-dimensional flow

$$\frac{n}{P' \frac{t_0}{t_e}} = \frac{A}{\left(\frac{t_e}{t_0} \right)^K M_e^{B-1}} \int_0^{\frac{x}{L}} \left(\frac{t_e}{t_0} \right)^K M_e^{B-1} d\left(\frac{x}{L} \right) \quad (2)$$

while for axially symmetric flow

$$\frac{n}{P' \frac{t_0}{t_e}} = \frac{A}{\left(\frac{t_e}{t_0} \right)^K M_e^{B-1} R^2} \int_0^{\frac{x}{L}} \left(\frac{t_e}{t_0} \right)^K M_e^{B-1} R^2 d\left(\frac{x}{L} \right) \quad (3)$$

Reference 1 suggests that for moderate pressure gradients the coefficient A be set equal to 0.44 and the coefficient B be chosen from a figure which is presented herein as figure 1. If an expected favorable pressure gradient is consistently that corresponding to stagnation-point flow or stronger, the coefficients A and B should be evaluated from the appendix. Such large pressure gradients might be encountered, for example, on the nose portions of blunt bodies or in choked nozzles.

Thus, all quantities in equation (1) are determined directly by the Mach number and static-temperature distributions with the exception of the shear or skin-friction parameter l and the Reynolds analogy parameter $\left(\frac{C_f Re_w}{Nu} \right)_{Pr=1}$. These parameters are presented in reference 1 as functions of the correlation number n . In order to complete the calculation of heat transfer according to reference 1, the parameter n must be evaluated from the quantity $n/P' \frac{t_0}{t_e}$, which requires differentiation of the

pressure distribution; the parameters l and $\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}$ are then obtained and substituted into equation (1). However, as can be seen from figure 2, the ratio $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$, which is the desired combination of

parameters, is relatively insensitive to n for many cases of interest. Hence, it is suggested that $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$ be taken as 0.11 for all values

of n .¹ This assumption gives an error of less than 10 percent except for flows with large favorable pressure gradients or very near separation. For large favorable pressure gradients, a further suggestion is made in the appendix with regard to the quantity $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$.

The expression for heat transfer upon setting $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$ equal to 0.11 is

$$\left(\frac{Nu}{\sqrt{Re_w}}\right) = 0.22 Pr^\alpha \sqrt{\frac{\frac{x}{L}}{\frac{n}{P' \frac{t_0}{t_e}}}} \quad (4)$$

where $n/P' \frac{t_0}{t_e}$ is obtained from either equation (2) or (3). The quantity Pr^α accounts for the effect of Prandtl number on heat transfer. Values of α suggested by Tifford and Chu (ref. 6) are, for small pressure gradients, $1/3$; for large adverse pressure gradients, $1/4$; and for extreme favorable pressure gradients, $1/2$.

It should be noted and emphasized here, that if skin-friction information is desired, the parameter l must be evaluated, and, therefore, the complete method of reference 1 must be used.

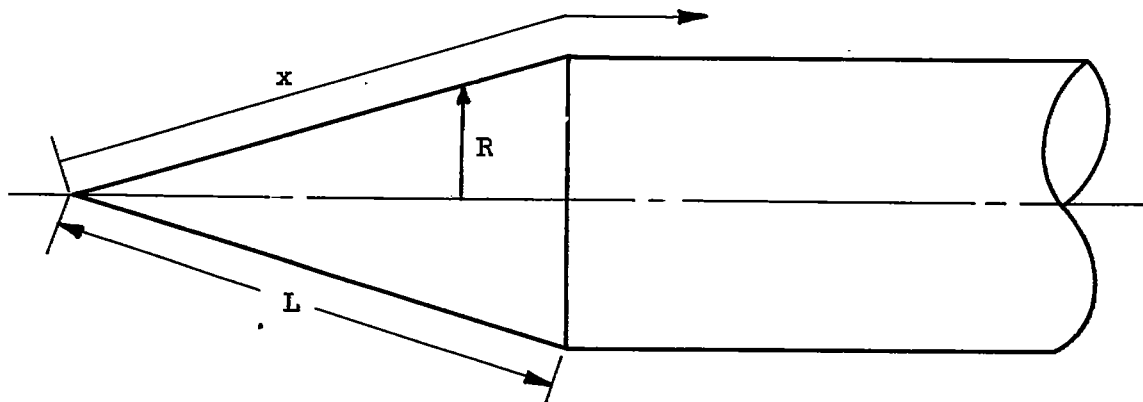
¹This same approximation can be obtained from the almost linear variation of a heat-transfer parameter r/S_w with correlation number n shown in fig. 6 of Crocco and Cohen (ref. 5). The present approximation corresponds to $r/n S_w = -0.22$.

EXAMPLES

Three examples are given. The first two are a cone cylinder and a parabolic-nose cylinder for which Jack and Diaconis (ref. 4) recently presented experimental information. The numerical calculations for these two examples are for a free-stream Mach number of 3.12, ambient static temperature of 180° R, and for the surface cooled to ambient static temperature. The third example is that of the rocket nozzle calculated in reference 1.

I. Cone Cylinder

Because the cone cylinder is composed of two simple shapes, it lends itself to a somewhat generalized treatment. The length of the cone surface will be designated L as shown in the following sketch:



Over the conical portion of the body the surface Mach number is constant so that from equation (3)

$$\left(\frac{n}{P' \frac{t_0}{t_e}} \right)_{\text{cone}} = \frac{0.44 \left(\frac{x}{L} \right)}{3} \quad (5)$$

and from equations (4) and (5)

$$\left(\frac{Nu}{\sqrt{Re_w}} \right)_{\text{cone}} = 0.575 Pr^{1/3} \quad (6)$$

in which the coefficient 0.575 is identically that indicated by the exact boundary-layer solution for a Prandtl number of 1.

On the cylindrical portion of the body from equation (3),

$$\frac{n}{P} \frac{t_0}{t_e} = \frac{0.44}{M_e^{B-1} \left(\frac{t_e}{t_0}\right)^K} \left[\frac{M_{e\text{ cone}}^{B-1} \left(\frac{t_e}{t_0}\right)_{\text{cone}}^K}{3} + \int_1^{\frac{x}{L}} M_e^{B-1} \left(\frac{t_e}{t_0}\right)^K d\left(\frac{x}{L}\right) \right] \quad (7)$$

and the heat-transfer parameter $Nu/\sqrt{Re_w}$ is obtained by substituting equation (7) into equation (4).

The heat-transfer rate on the cylinder side of the cone-cylinder junction can be readily evaluated. From equations (4) and (7),

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{\text{cyl at junction}} = \left(\frac{Nu}{\sqrt{Re_w}}\right)_{\text{cone}} \sqrt{\frac{M_{e\text{ cyl}}^{B-1} \left(\frac{t_e}{t_0}\right)_{\text{cyl}}^K}{M_{e\text{ cone}}^{B-1} \left(\frac{t_e}{t_0}\right)_{\text{cone}}^K}} \quad (8)$$

Equations (5) to (8) are used to calculate the heat transfer to the 90° -included-angle cone cylinder reported in reference 4. The pressure distribution for this cone cylinder, shown in figure 3, was obtained using the theory of Van Dyke (ref. 7). An examination of the calculated values of $Nu/\sqrt{Re_w}$ shows that the presented approximate technique gives results within 2 percent of those using the complete method of reference 1. The abrupt change in the heat-transfer parameter at the cone-cylinder junction is due to the abrupt break in the pressure and Mach number distribution at this station.

Jack and Diaconis (ref. 4) have proposed that the heat transfer on the cylindrical portion of the body be estimated by a formula which matches the cylinder value at the cone-cylinder junction and the flat-plate value at infinity. This relation takes the form (for $x > L$)

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{\text{cyl}} = \frac{\left(\frac{Nu}{\sqrt{Re_w}}\right)_{\text{cone}}}{\sqrt{3} \sqrt{1 - \frac{3C^2 - 1}{3C^2} \frac{L}{x}}} \quad (9)$$

where

$$C = \frac{\left(\frac{Nu}{\sqrt{Re_w}} \right)_{\text{cyl at junction}}}{\left(\frac{Nu}{\sqrt{Re_w}} \right)_{\text{cone}}}$$

as evaluated from equation (8). The use of equation (9) avoids the necessity of performing the integration of equation (3) over the cylindrical portion of the body, and it is seen from figure 3 that equation (9) agrees closely with the other estimations of heat transfer on the cylindrical portion of the body. All three curves in figure 3 represent the data of reference 4 to within 15 percent.

II. Parabolic Nose Cylinder

A sketch of the parabolic nose cylinder together with its pressure distribution as calculated using the theory of reference 7 is shown in figure 4. The distribution of heat-transfer parameter $Nu/\sqrt{Re_w}$ as calculated by the present approximate technique using equations (3) and (4) is seen to be within 3 percent of that obtained using the complete method of reference 1.

A further simplification for this slender parabolic-nosed body is proposed in reference 4. If it is assumed that the Mach number over the body is fairly constant and that the heat-transfer variation is due primarily to the geometric variation of the body from conical to cylindrical shape, then from equations (1) and (3)

$$\frac{\frac{Nu}{\sqrt{Re_w}}}{\left(\frac{Nu}{\sqrt{Re_w}} \right)_{\text{flat plate}}} = \sqrt{\frac{R^2 x}{\int_0^x R^2 dx}} \quad (10)$$

The curve of equation (10) is seen from figure 4 to be in good agreement with those of the complete (ref. 1) and simplified techniques. All three curves represent the experimental data of reference 4 within 12 percent.

III. Rocket Nozzle

The third example is that of the rocket nozzle (fig. 5) also used as an example in reference 1. It is an example involving large favorable

pressure gradients with heat transfer. The combustion-chamber stagnation pressure is assumed to be 500 pounds per square inch absolute, the stagnation temperature is taken as 4000° R, the Prandtl number is assumed to be 0.78, and the ratio of specific heats is 1.3. The nozzle wall is assumed cooled to a uniform temperature of 800° R, which corresponds to $t_w/t_0 = 0.2$.

The solid line in figure 5 is the variation of the heat-transfer parameter $Nu/\sqrt{Re_w}$ from the complete method of reference 1. The simplified technique was applied in two ways: first, as proposed in the text of this report and second, as proposed for large favorable pressure gradients in the appendix. The results are shown in figure 5. While both applications of the simplified technique result in an overestimation of the heating lead, the technique for large pressure gradient matches the result of reference 1 at the stagnation point and overestimates the heat-transfer rate at the throat only by about 13 percent.

CONCLUDING REMARKS

An approximation is made in the method of Technical Note 3326 which simplifies the calculation of heat transfer.

The simplified method is expected to be adequate for isothermal surfaces with adverse and small favorable pressure gradients regardless of surface temperature level and also for flow with large favorable pressure gradient over cooled surfaces.

Good agreement is obtained in these ranges between the simplified technique and the complete method of Technical Note 3326 for the illustrative examples.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, September 13, 1956

APPENDIX - CALCULATION PROCEDURE FOR LARGE FAVORABLE PRESSURE GRADIENTS

The simplified technique as proposed in the text of this report is set up in such a way as to match the exact boundary-layer solution for zero pressure gradient. For large favorable pressure gradients (corresponding to stagnation-point flow or stronger), the use of the values of A, B, and $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$ proposed in the text may greatly overestimate the amount of aerodynamic heating.

The values of A and B in the following table correspond to lines tangent to the curves of figure 3 of reference 1 at the plane stagnation point. Similarly, the values of $\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$ and α are for plane stagnation point flow.

$\frac{t_w}{t_0}$	A	B	$\frac{l}{\left(\frac{C_f Re_w}{Nu}\right)_{Pr=1}}$	α
0	0.380	2.35	0.102	0.4
.2	.372	2.53	.099	.4
.6	.362	3.20	.093	.4
1.0	.410	4.94	.083	.4

The simplified technique presented in this report is not recommended for use with large favorable pressure gradients over heated surfaces.

While the numbers presented in this appendix are for matching at plane stagnation-point flow, a similar match could be made at any desired condition using the information in figure 3 of reference 1 and in figure 2 of this report.

REFERENCES

1. Cohen, Clarence B., and Reshotko, Eli: The Compressible Laminar Boundary Layer with Heat Transfer and Arbitrary Pressure Gradient. NACA TN 3326, 1955.
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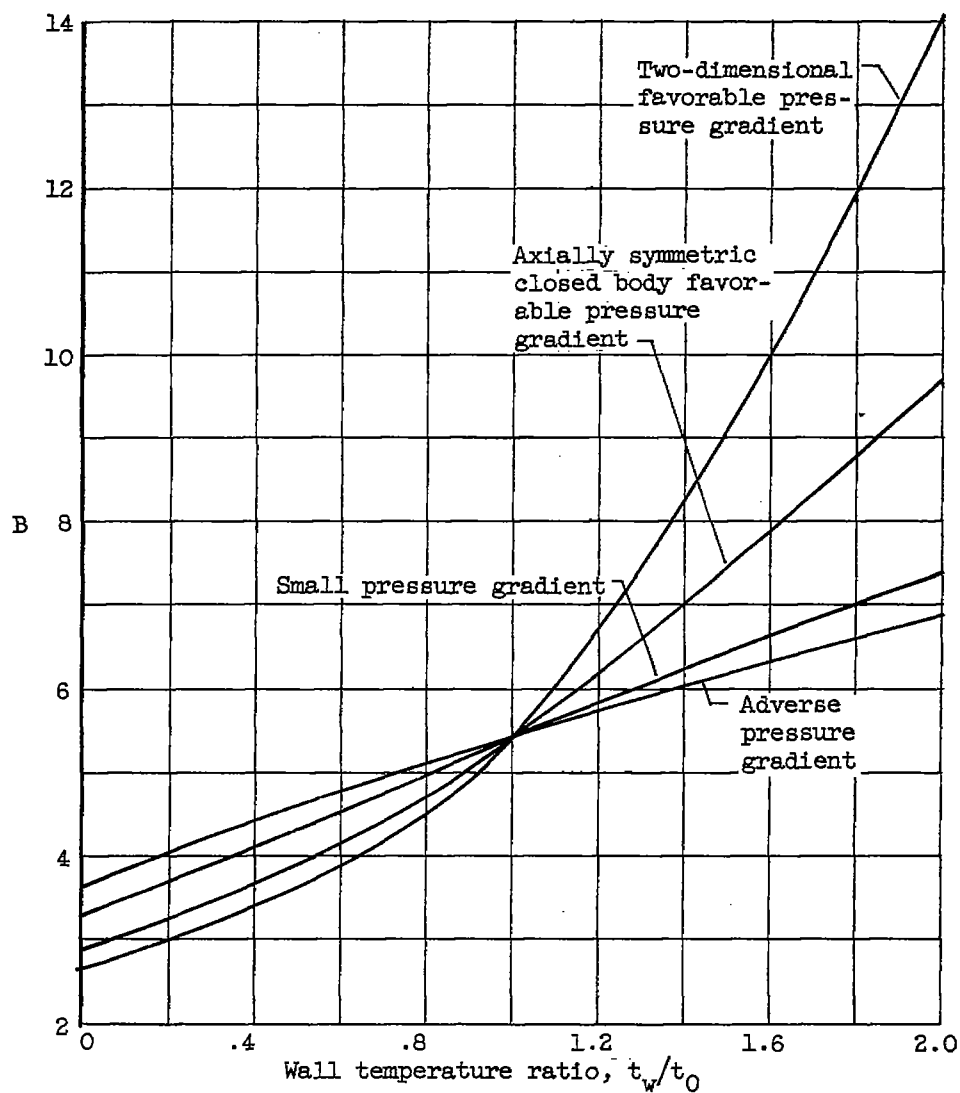


Figure 1. - Values of exponent B to be used in equations (2) or (3). (From ref. 1.)

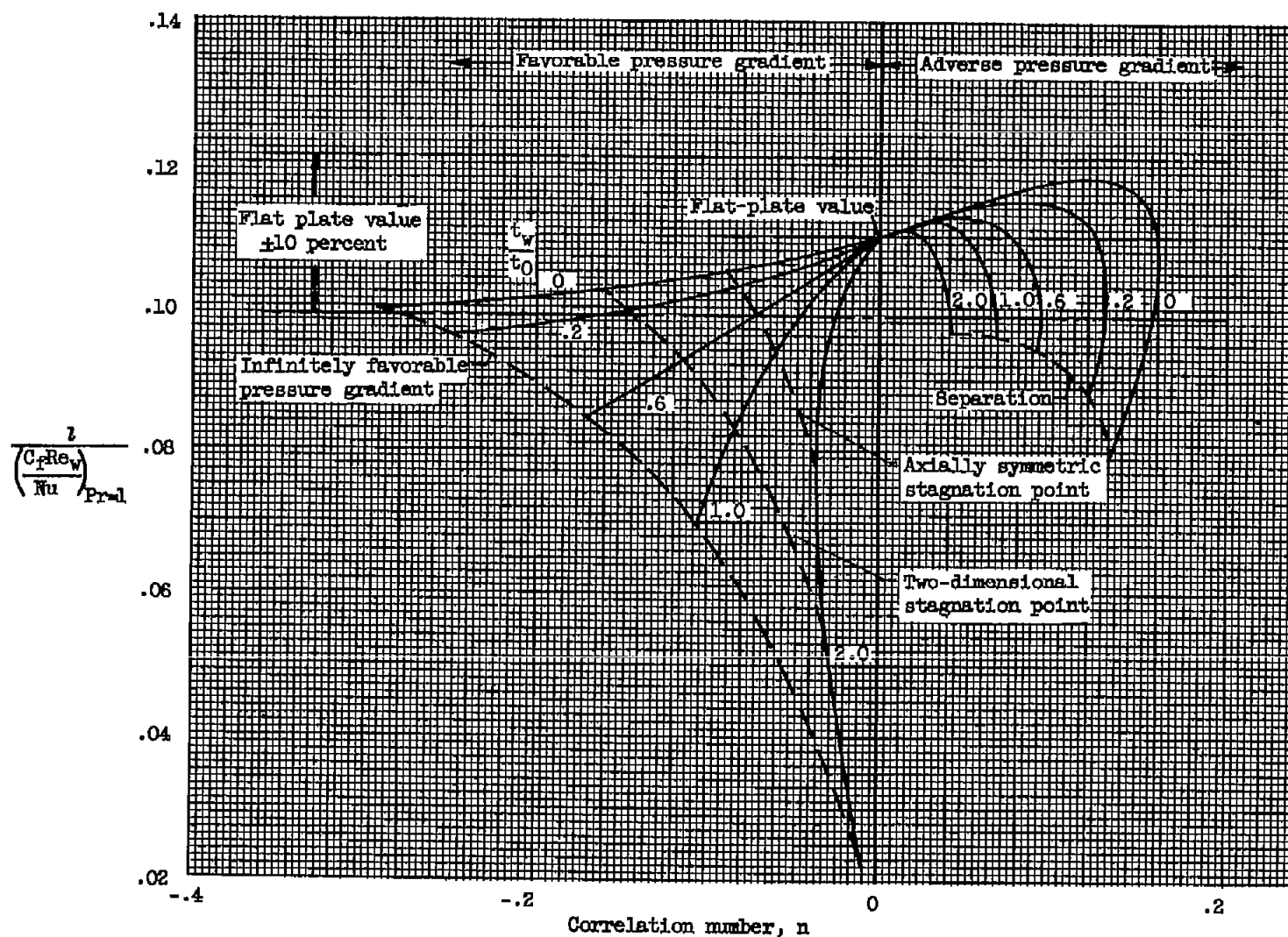


Figure 2. - Ratio of shear parameter to Reynolds analogy parameter from solutions reported in reference 1.

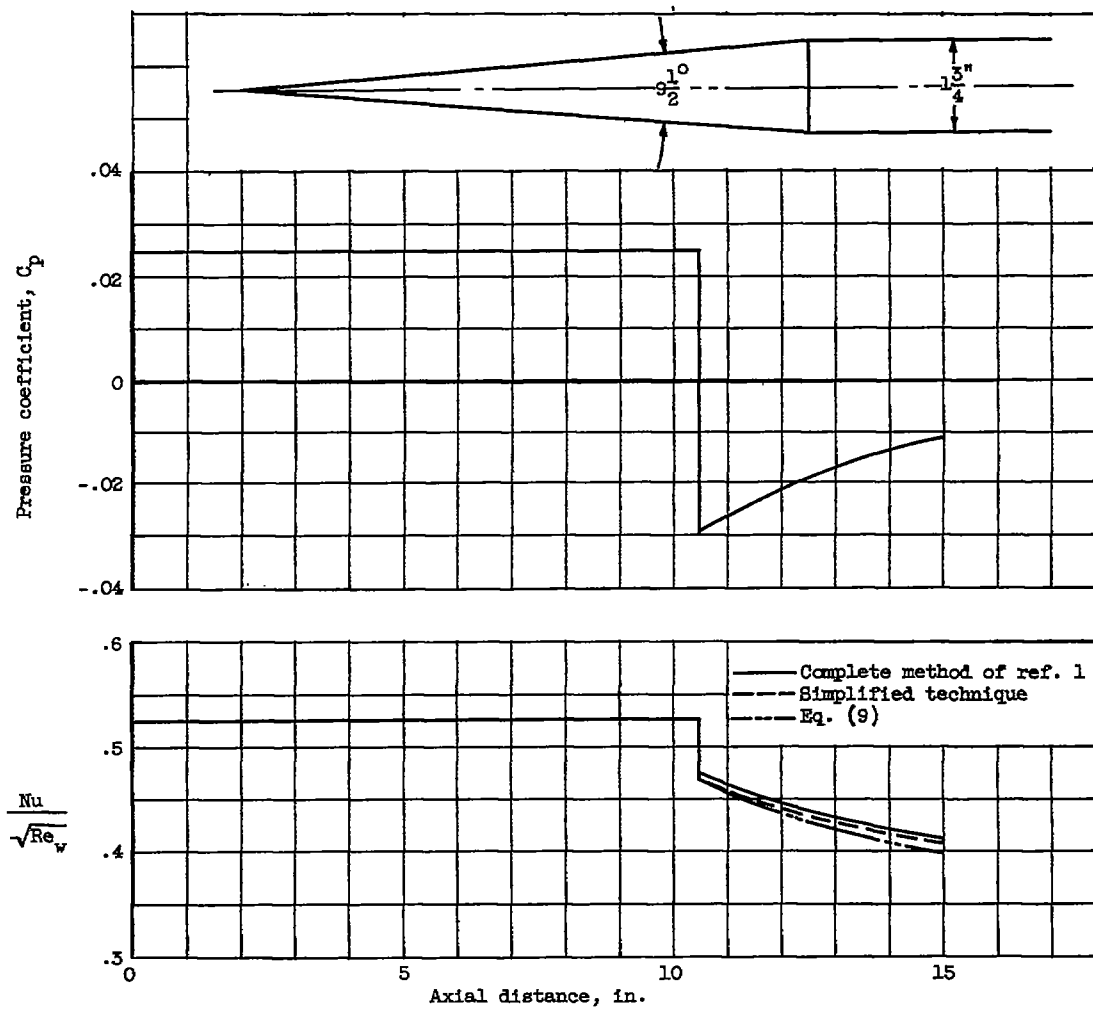


Figure 3. - Example I: Cone cylinder body.

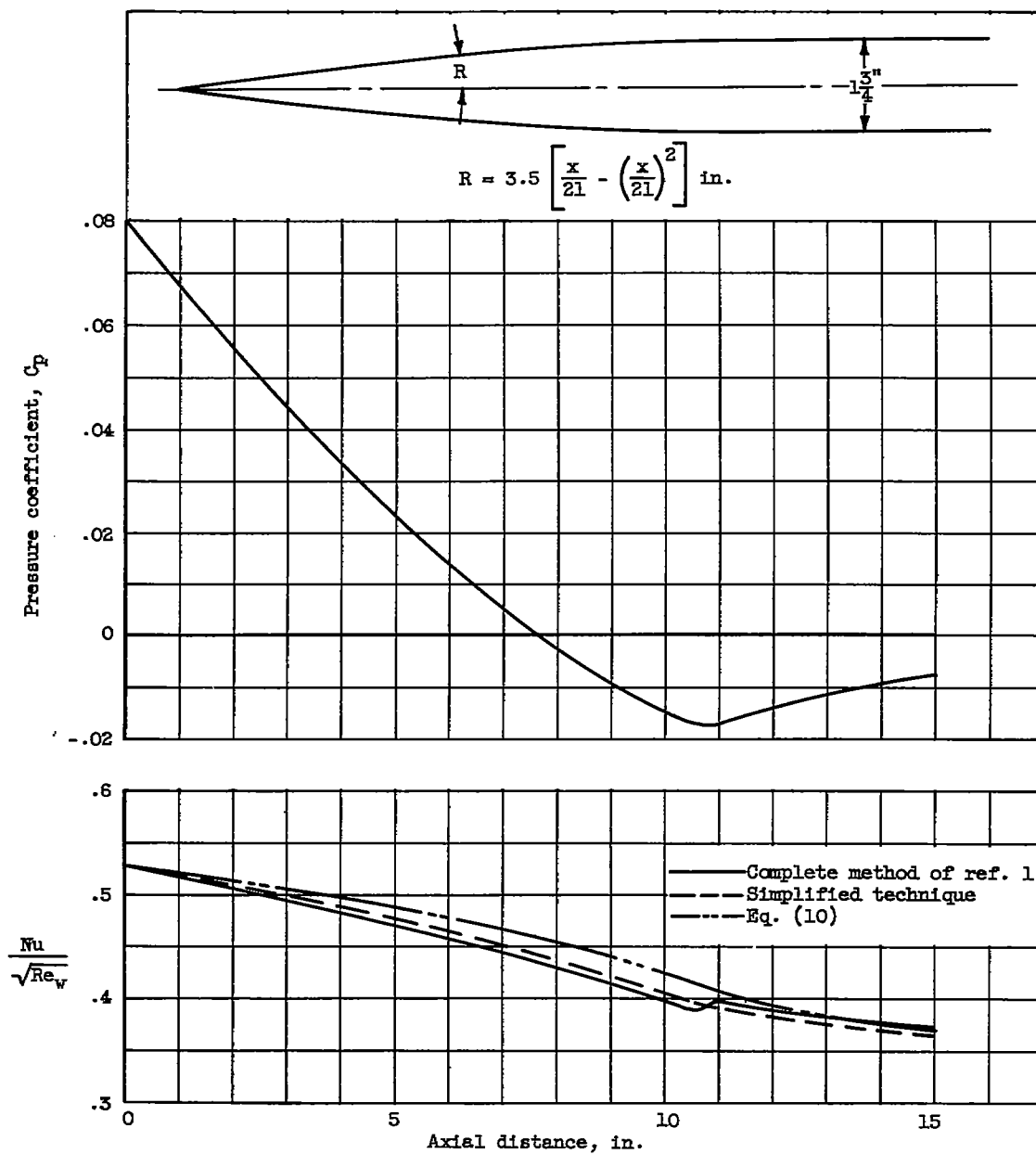


Figure 4. - Example II: Parabolic-nose-cylinder body.

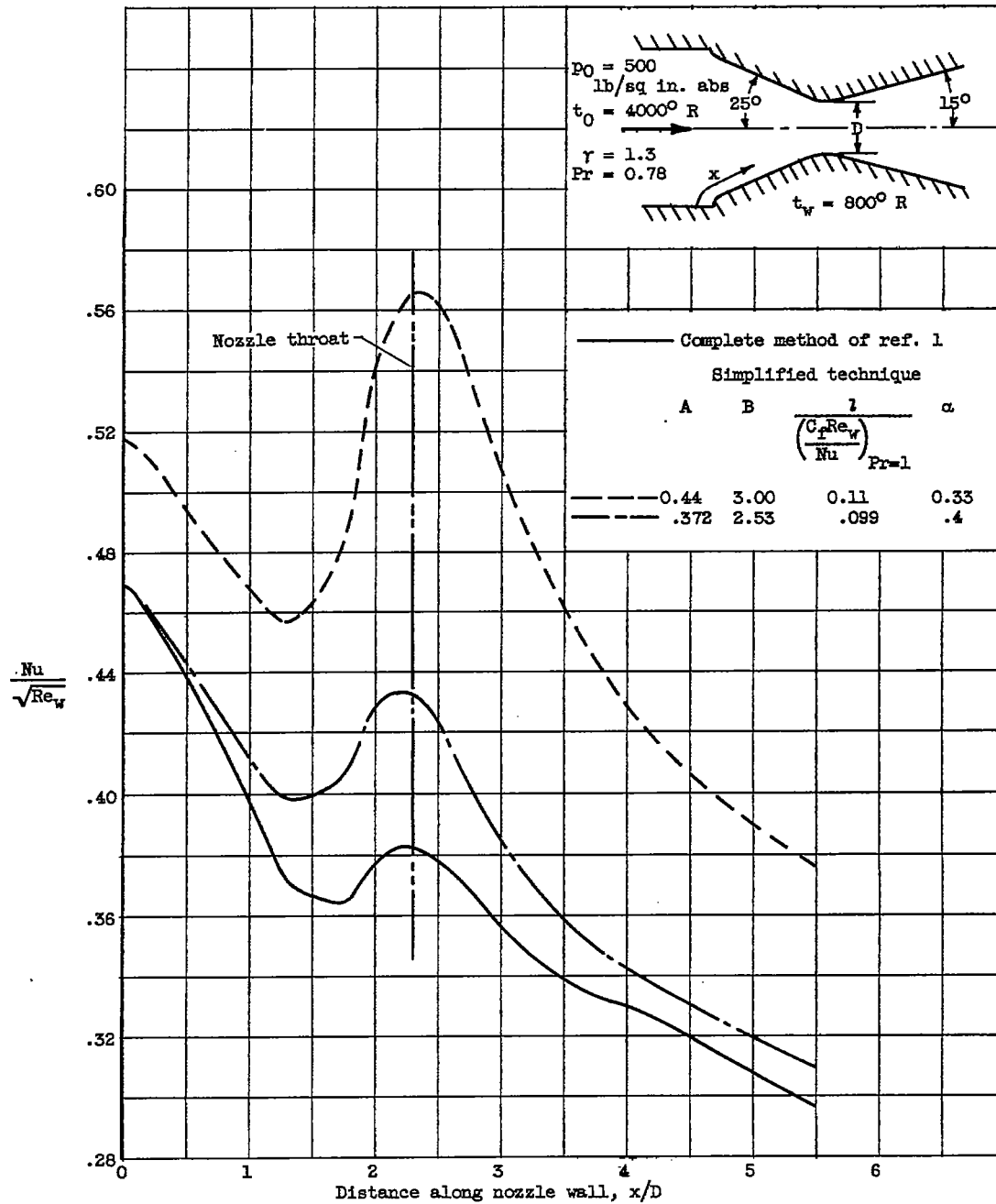


Figure 5. - Example IV: Rocket nozzle